Zeta Distribution and Transfer Learning Problem

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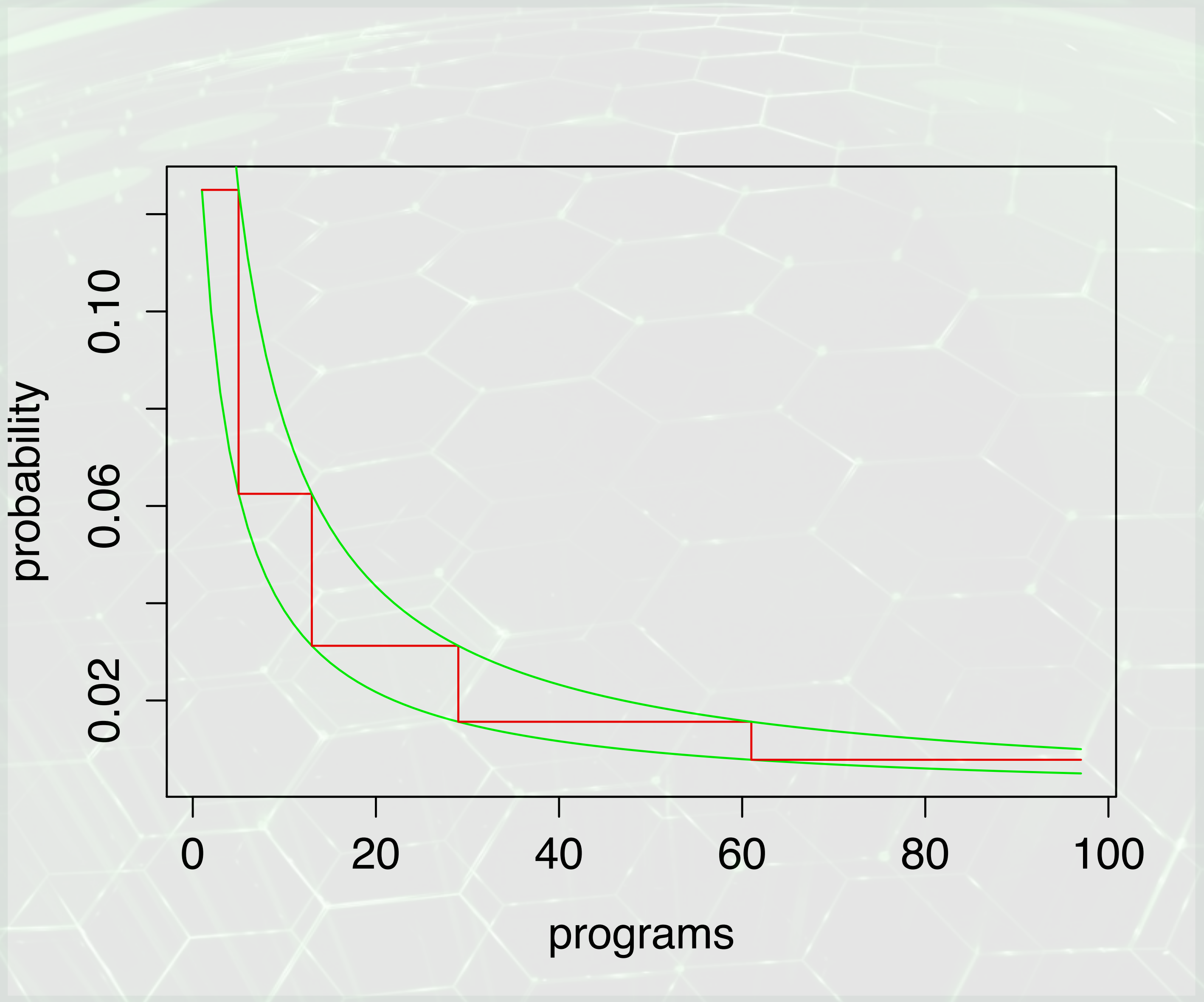
Introduction

- Two types of AI research
  - Constructive AI
  - Analytical AI
- Why is human learning effective?
  - The world is hierarchical
  - Physics results in simple forms: unitary evolution in QM
  - The world is evolutionary
- Goal:
  - To argue that an evolutionary world makes transfer learning feasible.
  - We show two unrealistic learning models that are likely not feasible.
  - We show two evolutionary learning models that are likely feasible.
- Zeta distribution can help model transfer learning

$$(2a)^{-1} \leq 2^{-\lceil \log_2 a \rceil} \leq a^{-1}, \text{ for } a \geq 4$$



Sandwich Theorem

» A sandwich theorem showing that an arithmetization of programs can be used for approximating a priori program probabilities.

Zeta Distribution of Programs

- »Arithmetization: $\phi(\pi) = \sum_{i=1}^{i \leq |\pi|} b_i . 2^{|\pi|-i}$
  - »interpret program bit strings as integers.
- »Zipf dist: $P(Z_s^{(n)} = o_i) \triangleq \frac{1}{i^s H_{n,s}}$
- »Zeta dist: $P(Z_s = k) = \frac{1}{k^s . \zeta(s)}$ Zeta fun: $\zeta(s) = \sum_{n=1}^{\infty} \frac{1}{n^s}$
- »Approximation: $P_M(x) \approxeq \sum_{M(\pi)=x*} \frac{1}{(\phi(\pi)+1)^{1+\epsilon} . \zeta(1+\epsilon)}$

Training Sequence as a Stochastic Process

- We extend Solomonoff's model to a stochastic process:
  - $\mathcal{M} = \{\mu_1, \mu_2, \mu_3, \ldots, \mu_n\}$
- Entropy rate: $H(\mathcal{M}) = \lim_{n\to\infty} \frac{H(\mu_1, \mu_2, \mu_3, \ldots, \mu_n)}{n}$
- Cond entropy rate: $H'(\mathcal{M}) = \lim_{n\to\infty} \frac{H(\mu_n | \mu_1, \mu_2, \mu_3, \ldots, \mu_{n-1})}{n}$
- Kolmogorov-Shannon theorem:
  - $\lim_{n\to\infty} \frac{K_M(X_1, X_2, X_3, \ldots, X_n)}{n} = H(X) + O(1)$
- Optimal program:
  - $\pi_i^* = \arg\min_{\pi_j}(\{|\pi_j| \mid \forall x, y \in \{0,1\}^* : M(\pi_j, x, y) = P(\mu_i = x | y)\})$
- Assume transfer learning oracle

Expected Time

- Conditional algorithmic entropy rate:
  - $K'(\mathcal{M}_{<k}) \triangleq K(\mu_k | \mu_1, \mu_2, \mu_3, \ldots, \mu_{k-1})$
- Conditional stochastic process probabilities:
  - $P'(\mathcal{M}_{<k}) \triangleq P(\mu_k | \mu_1, \mu_2, \mu_3, \ldots, \mu_{k-1})$
- Expected time:
  - $\mathbb{E}'[t(\mathcal{M}_{<k})] \triangleq \mathbb{E}_{\mathcal{M}}[t(\mu_k) | \mu_1, \ldots, \mu_{k-1}] \leq \sum_{\forall \mu_k \in \{0,1\}*} 2t(\pi_k^*) 2^{K'(\mathcal{M}_{<k})} P'(\mathcal{M}_{<k})$

Power-Law in Nature

- Preferential attachment in evolutionary systems (Yule)
- It follows from the principle of maximum entropy where mean of logs of observations is fixed (Visser)
- Gene family sizes vs. frequencies follow power law (Huynen)
- Gene expression in various species follows Zipf's law (Furusawa)
- Log-normal distribution of evolution rate of orthologous genes. Power-law like distribution of paralogous family sizes and network node degree. (Koonin)

Random Typing Model

- »Monkeys typing m-bit random programs
- »No mutual information
- »Oracle achieves no time saving
- »Compatible with:
  - »Levin's conjecture that AI is impossible
  - »No-Free Lunch Theorem sort of arguments
- »Incompatible with observation:
  - »Common sense knowledge in AI is useful

Identical Zeta Random Variables

- Process generated i.i.d. from zeta distribution
  - $H'(\mathcal{M}) = H(\mu_1) = H(Z_s)$
- Expected running time:
  - $\mathbb{E}'[t(\mathcal{M}_{<k})] \leq \frac{2t_{max}}{\zeta(s)} \sum_{k=1}^{\infty} 2^{\lceil \log_2 k \rceil} k^{-s} \leq \frac{4t_{max}}{\zeta(s)} \sum_{k=1}^{\infty} \frac{k}{k^s}$
- First trillion programs, s= 1.001:

$$t_{max} \sum_{k=1}^{10^{12}} 4k/k^{1.001} \zeta(1.001) \approxeq 3.89 \times 10^9 t_{max}$$

Zipf Distribution of Sub-programs

- Each program has m instructions:
  - $A = \{a_1, a_2, a_3, \ldots, a_{2^k}\}$
- Each optimal program made up of instructions:
  - Database of sub-programs:

$$\pi_i^* = \pi_{i,1}^* \pi_{i,2}^* \pi_{i,3}^* \ldots \pi_{i,m}^*$$

$$P^* = \pi_{i,j}^*$$

- Total entropy:
  - $H(\mu_1, \mu_2, \ldots, \mu_n) \approx \log_2 k + k.2^k + \log_2 n + \log_2 m + H(Z_s^{(2^k)})$
- Expected time:

$$H'(\mathcal{M}) \approx \lim_{n \to \infty} \frac{1}{n} \Big(k.2^k + \frac{s}{H_{2^k,s}} \sum_{l=1}^{2^k} \frac{\ln(l)}{l^s} + \ln(H_{2^k,s}) + \log_2 k + \log_2 n + \log_2 m$$

An Evolutionary Zeta Process

- Random mutations of programs in training sequence:
  - $\pi_1^* = \wedge$

$$\pi_i^* = \begin{cases} M(Z_s, \pi_{i-1}^*), & \text{if } Z_s \text{ is a valid transformation} \\ \pi_{i-1}^*, & \text{otherwise} \end{cases}$$

- Small conditional entropy rate:

$$\lim_{n\to\infty} H'(\mathcal{M}) = H(Z_s) = \log(\zeta(s)) - \frac{s\zeta'(s)}{\zeta(s)}$$

- Calculation of conditional entropy rate:

$$H(Z_{1.1}) = 13.8 \qquad H(Z_{1.05}) = 24.5$$

$$H(Z_{1.01}) = 106.1 \qquad H(Z_{1.001}) = 1008.4$$

Questions?

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