

# On Effective Causal Learning

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**Abstract.** We have developed a framework for identifying causal relationships between events which are *effective* in the sense that they can be put to practical use, without regard to what the “true” causes really are. A rapid causal learning process is devised for temporally correlated events that can be observed proximally which is sufficient for the learning of many causalities involving basic physical and social phenomena. The system relies on a *diachronic* aspect of causes which is a characterization of consistent temporally correlated events and a *synchronic* aspect of causes which is a characterization of the contextual factors that enable the diachronic causal relations. The causal learning method is applied to some problem solving situations that allows some basic knowledge to be learned rapidly and that results in drastic reductions of the search space and amount of computation involved. This method is necessary to jump start the chain of causal learning processes that allow more complex and intricate causal relationships to be learned based on earlier knowledge.

## 1 Introduction

Being able to establish causality is critical to a cognitive system’s survival and proper functioning and is the most important foundation upon which a cognitive system structures its intelligent behavior. However, various research seems to suggest that there is no simple algorithm that is available that can easily and quickly establish causality between two events [1, 2, 3]. One key problem is as follows. Suppose a correlation is detected between two events A and B. It can be argued (and often it may be the case) that there exists a yet to be observed third event, C, that causes A and B separately and hence the correlation is not due to A causing B directly. A celebrated example is the issue of whether smoking causes lung cancer. Even if a correlation between smoking and cancer is established, one can argue (and many a tobacco company had raised the point) that perhaps there is a gene that causes people to like smoking and that it also causes lung cancer [2]. Often, domain specific knowledge needs to be brought to bear for the establishment of causality [1, 2]. However, there is a chicken-and-egg problem: the learning and acquisition of some commonsensical domain knowledge which encodes fundamental rules/models of the world, both physical and social, requires the establishment of causality between some fundamental physical events (e.g., push something and it will move) or social events (e.g., yell out loud and everyone will turn to look at you immediately) to start with. Therefore, by appealing to domain knowledge only begs the question of the origin of this knowledge.

In our previous papers [3, 4] we proposed a paradigm of research in which an adaptive autonomous agent would, starting from no prior knowledge, interact with the environment and extract causal rules of behavior between physical/social causes and physical/social objects on an ongoing and rapid basis. How is this achievable?

We propose a method that is *sufficient* to allow some very fundamental causality – e.g., those involving physical laws – to be learned. These acquired commonsensical domain knowledge could then support the subsequent learning of increasingly complex and intricate causalities. We then illustrate how the application of this basic causal learning process can vastly reduce the computational effort involved in solving two AI problems – the “spatial movement to goal” and “crawling robot” problems [6].

## 2 Motivational Considerations

Important insights can be gained on the issue of causality by returning to the very root of what causality means and what it can do for us. Consider a simple example of, say, using a remote control to control a toy car. Suppose this pair of objects is given to a child or is picked up by a primitive person in the jungle, both of whom have no prior domain knowledge on the devices. By playing with the joystick on the remote control and observing the effects it has, say, on the wheels of the toy car, they can easily and quickly establish the causality between a certain movement of the joystick (an event) and the way the wheels turn (another event) based on the temporal correlation observed, much like they would learn very quickly about lightning and thunder. Someone observing them carrying out the acts can also easily observe the correlation and hence deduce the causality.

However, one can still posit the possibility that there is a third event, perhaps an action of “God,” that causes the person to push the joystick on the remote control in a certain direction and also causes the turning of the wheel of the car accordingly every time. Therefore, strictly speaking, one still cannot conclude that the person’s certain action is the *true cause* of a certain turning behavior of the wheel. How then can a cognitive system even begin to learn some very fundamental causalities to start with?

We believe it is useful to define a concept called *effective causality* that supports the learning of fundamental causal knowledge. In the case of the remote control and the car, in the absence of other observable possible causes (such as a “God”), it can be assumed that if there is a consistent temporal relationship between the joystick event and the wheel event, the joystick event is the *effective cause* of the wheel event. Whether there is an underlying “true cause” such as an unobservable “God” does not matter. The effective causal rule serves a practical purpose for the system to be able to predict the consequence of its action. Of course, the rule may cease to be valid later which is fine because the system then learns other new rules rapidly.

Similarly, in the case of smoking and lung cancer, if one’s purpose is a practical one of, say, avoiding a mate who has a high chance of getting cancer, then the existence of the correlation is sufficient, whether smoking is “really” the cause does not matter, unless one’s purpose is to establish a case against the tobacco companies.

The correlation between the remote control and toy car is also a lot easier to detect than that between smoking and lung cancer. This is because the remote control and toy car's behaviors are observed by the person (or the observer observing the person controlling the remote control) directly in a proximal manner. Provided their senses can be trusted, the correlation is easily established. Of course, there could be occasional noise – such as the coincidental movements of other objects around - but they can be weeded out fairly quickly by one or two more tries at the pushing of the joystick in the same direction – these coincidental events are not likely to recur. Thus, in the case of smoking and lung cancer, if an observer is able to observe the interaction of the smoke particles and the lung cells directly, the correlation would be much more easily established. Otherwise, more complex statistical analysis is needed [2, 3].

We term the effective cause of an event arising from temporal correlation a *diachronic* cause. There is yet another aspect of effective causality: there are “enabling” contextual *factors* that enable the temporal correlation between two events, without which the temporal correlation will not exist. E.g., when *on Earth* the law of gravity holds – that an object, when let go (an event), will fall to the ground in the next instance (another event). We identify these factors as *synchronic* causes.

We shall consider the example of gravity more closely in connection with the identification of synchronic causes. Actually when humans first encountered the phenomenon of gravity, because it took place in practically any context/background on Earth, our original conception was that it was ubiquitous and the background did not matter. Hence we first characterized gravity as applicable *everywhere*. Later we discovered that there are situations in outer space in which gravity can become reduced or non-existent. We then modified the concept of gravity to be applicable everywhere *on Earth* (the context) in its full strength – we bring back the context/background that we have ignored earlier. Therefore we happily applied the *effective* causal rule (that all objects would fall when not supported, in any context) for most of humanity's existence, using it to successfully support our survival until a new rule was discovered. This is the way we will handle the discovery and characterization of synchronic causes – the enabling contextual factors to be described in Section 3.2. The physicist Lee Smolin [7] believes that all physical theories are *effective* and approximate that apply to “truncations” of nature and that cover only a limited domain of phenomena. This echoes our idea of effective causality.

### **3 An Effective Causal Learning Framework**

In the spirit of the discussion above, we divide the identification of causes into the two major aspects: the *diachronic* and the *synchronic* aspects. For the current paper, we consider only deterministic situations in which a diachronic cause must always bear a consistent temporal relationship with the effect and a synchronic cause must always be present. Considerations of probabilistic situations are relegated to a future paper. Our focus in this paper would be to show the application of the basic causal learning framework which is *sufficient* to drastically reduce search efforts in some problem solving situations.

### 3.1 The Identification of Diachronic Causes

We begin by considering a simple situation in which a few simple events (changes of states) can be directly (proximally) observed to take place as shown in the left diagram in Fig. 1. These events consist of Appearances and Disappearances of the three objects 11, 12, and 13 at some locations L1, L2, and L3 respectively which we will represent as  $\text{App}(\text{object}, \text{location})$  and  $\text{Disapp}(\text{object}, \text{location})$  respectively. In the right of Fig. 1 is a temporal diagram in which the horizontal axis is time represented discretely as a succession of “time frames”  $t_1, t_2, \dots$  etc. and the vertical axis lists the presence of the objects with the associated parameters - the locations at which they appear when they appear - at the corresponding time frame. So, say, in  $t_1$  there is no object present and in  $t_2$  object 11 appears at L1. The change from  $t_1$  to  $t_2$  is noted (as a gray vertical bar). (An event could be the change of the existential or other states of an object or the change of some parameters such as the location or the energy level associated with an object - e.g.,  $\text{Move}(11, L1, L2)$ : object 11 moves from location L1 to location L2). In general, whenever there is a change, the system would look for a “cause” – some changes that happened earlier in time. However, because this is the first change since the beginning of “time” in this “mini-universe,” it is assumed that either there is no cause or the cause is before the current temporal interval (e.g., a “first cause” such as “God” that exists outside the current space-time continuum.)

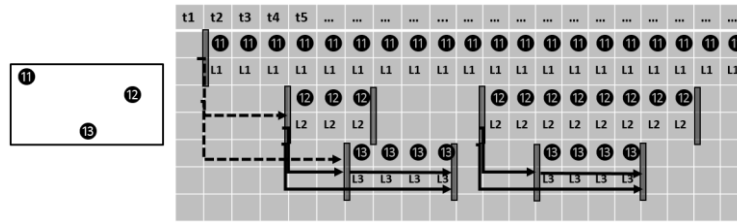


Fig. 1. Diachronic causes. See text for explanation.

Henceforth we will refer to a *causal rule* as one that links an event as a *diachronic* cause (e.g.,  $\text{App}(11, L1)$ ) and another event as a *diachronic* effect (e.g.,  $\text{App}(12, L2)$ ), denoted as  $\text{App}(11, L1) \rightarrow \text{App}(12, L2)$ , if such a causal connection exists. Looking at Fig. 1, one can see that there is a consistent correlation between  $\text{App}(12, L2)$  and  $\text{App}(13, L3)$  (with a delay of 2 time frames) because it happens twice. We require at least two consistent correlations before we construct the causal rule in order to weed out noise – this is called “dual instance consideration.” (In a probabilistic formulation, we can define “rule-confidence” in terms of the number of consistent instances observed.) Therefore,  $\text{App}(11, L1) \rightarrow \text{App}(12, L2)$  and  $\text{App}(11, L1) \rightarrow \text{App}(13, L3)$  do not hold as they each happens only once. There is also a causal rule  $\text{App}(13, L3) \rightarrow \text{Disapp}(13, L3)$  that holds that is a “self-causal” rule – it predicts that an object will disappear after a certain amount of time of its appearance (e.g., a timer controlling a light). But if that correlation exists, then the rule  $\text{App}(12, L2) \rightarrow \text{Disapp}(13, L3)$  will also follow as indicated in the figure. Without further knowledge or proximal observations (i.e., of a timer mechanism),  $\text{App}(12, L2)$  can be taken as the underlying cause

of *both* App(L3) and Disapp(L3) and the situation is similar to C being the underlying cause of the correlation of A and B discussed in the Introduction section.

The ability to weed out noise through dual instance consideration is dependent on how “busy” the environment is – how many events are happening in a given time period. If the scenario is very busy, say with millions of events happening in a small temporal interval, the number of instances involved may have to be increased to remove spurious correlations. But the dual instance consideration should suffice for our ordinary environment. Also, in the event that the cognitive systems is in a desperate need to look for a cause of something so as to be able to bring about or repeat an earlier observed effect, it may relax the dual instance consideration – it may try its luck and apply those temporal correlations that have been observed only once.

In the event that more than one diachronic causes are identified, they are encoded as *conjunctive diachronic causes*. Subsequent observations will identify whether they are really conjunctive causes – i.e., if one is subsequently observed to be absent and the effect does not ensue - or they are *disjunctive diachronic causes* – any one of them happening will give rise to the effect.

### 3.2 The Identification of Synchronic Causes

We consider the simple events in Fig. 2 (similar to Fig. 1) to illustrate the idea of synchronic causes as discussed above. In Fig. 2, the duration of the 11 event is shortened and a new 14 event is introduced as shown. Now, had object 11 been present “all the time” at location L1 as in Fig. 1, At(11, L1) would be considered a necessary *synchronic* (contextual) cause for App(12, L2)  $\rightarrow$  App(13, L3). (Currently there is another entity that is a synchronic cause of App(12, L2)  $\rightarrow$  App(13, L3), which is the location associated with App(12, L2) - At(12, L2)). In Fig. 2, however, because only *either* 11 *or* 14 is present at their corresponding locations at the time when App(12, L2) happens (shown in two dashed rectangles), the presence of both 11 and 14 are *tentatively* considered *not* to be the synchronic causes accompanying App(12, L2)  $\rightarrow$  App(13, L3), in the same spirit as the gravity example discussed in Section 2.

However, this tentative removal of the presence of 11 and 14 as synchronic causes of App(12, L2)  $\rightarrow$  App(13, L3) will be reverted as soon as another event 12 takes place (say, outside the time frame shown in Fig. 2) in the absence of 11 and 14, and 13 fails to take place after 2 time intervals. This shows that the presence of 11 *or* 14 were indeed needed for App(12, L2) to cause App(13, L3). The system then modifies the already learned causal rule App(12, L2)  $\rightarrow$  App(13, L3) to include At(11, L1) *or* At(14, L4) as its accompanying disjunctive synchronic causes, in conjunction with At(12, L2) (i.e., At(12, L2) *and* (At(11, L1) *or* At(14, L4))).

Suppose A, B, C, D, E, ... are synchronic causes, the above method can recover complex conjunctive/disjunctive combinations of these such as A and (B or C) or (D and E)... A dense collection of disjunctive synchronic causes that are parameter values (e.g. L1 = 1.1 or 1.2 or 1.3 or 1.4...) can be combined into a sub-range of values (e.g. 1.1 < L1 < 1.9). In the case of gravity discussed in Section 2, the disjunctive “sum totality” of each context (synchronic cause) on Earth in which gravity applies is the context of the *entire* Earth.

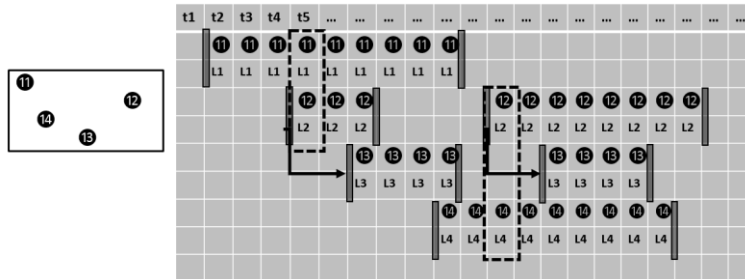


Fig. 2. Synchronic causes. See text for explanation.

## 4 Application of Causal Learning to Problem Solving

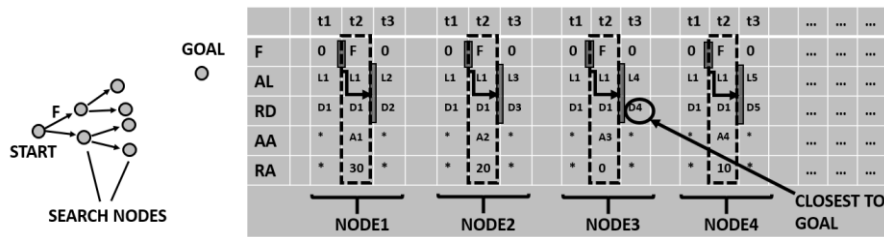
In this section, we discuss two examples in which the effective causal framework can be used to drastically reduce the amount of computation in problem solving situations. One example is the typical spatial movement to goal problem and the other is the crawling robot problem [6]. Typically, the A\* [8] or best-first search process is used for the spatial movement to goal problem and reinforcement learning is used for the crawling robot problem [6, 9]. In both cases, no *causal* characterizations of the actions involved are learned or extracted in the processes, resulting in relatively “blind” searching processes that require a large amount of computation. We show that our framework developed above discovers/learns causal rules of various actions involved in the search process that have general and wide applicability that can result in a drastic reduction of the search space involved.

### 4.1 The Spatial Movement to Goal Problem

Consider the spatial movement to goal problem - a cognitive system starting from a point in space and trying to reach a physical point (the GOAL) some distance away (Fig. 3). Using, say, the best-first search to solve the spatial movement to goal problem, a domain *independent* heuristic is typically used such as the “shortest distance to goal” heuristic. However, with this heuristic, one still needs to expand many nodes at each level of the search to compute the heuristic value in order to select the one with the minimum value for the next level of expansion. It is possible to use a domain *dependent* heuristic – always head straight toward the GOAL – and obviate the need to expand many nodes. However, building in this heuristic will be too contrived. Is there a domain independent method that allows the system to discover this domain dependent heuristic? Our effective causal learning algorithm described in Section 3 provides just such a domain independent mechanism to learn this piece of knowledge – a manner of applying an elemental force that *causes* the “shortest distance to the goal” to be achieved. (Here we assume that there are no obstacles between the cognitive system

and the goal. Our effective learning framework can also handle the situation with obstacles that we relegate to a future paper.)

Fig. 3 shows that in the first level of the best-first search process all the possible directions of the applied force,  $F$ , is tried and the parameters associated with these “expanded nodes” are kept track of. We assume that the parameters available to the “senses” of the cognitive system are: AL (its absolute location); RD (its relative distance to the GOAL – provided by, say, a vision system); AA (the absolute angle with respect to the entire reference frame in which the force is applied); and RA (the relative angle to the GOAL in degrees which is defined as 0 when the force is pointing directly at the GOAL). These parameters can potentially be the synchronic causes or diachronic effects accompanying the diachronic cause – the force. (At the moment the force is applied, if there are other objects around, there will also be parameters such as the relative distances and the force’s directions relative to these objects. However, given our current method, these factors would be ruled out quite rapidly within a few movements of the cognitive system and hence for simplicity without sacrificing generality, we omit these other parameters for the current discussion.)



**Fig. 3.** Search with effective causal learning.  $F$  = Force; AL = Absolute Location; RD = Relative Distance (to GOAL); AA; Force Absolute Angle; RA = Force Relative Angle (to GOAL)

In NODE1 the force goes from 0 (non-existent) to  $F$  (existent) and the parameters associated with the cognitive system – AL and RD – change in some manner after one time frame. (Assuming the force always takes one time frame to effect changes.) Those parameters that change are the diachronic effects of the force application. (AA and RA are undefined (“\*”) before and after the force application.) NODE3, in which the force  $F$  is applied with the parameter values AL = L1, RD = D1, AA = A3, and RA = 0, which is denoted as  $F(L1, D1, A3, 0)$ , corresponds to the shortest distance to the GOAL. This node is selected for expansion. (At this stage the parameter values L1, D1, A3 and 0 are potential synchronic causes for the force event.)

In the next level of expansion,  $F(L4, D4, A3, 0)$  would be the action that satisfies the shortest distance to goal heuristic. The causal learning algorithm then generalizes the required force to  $F(*, *, A3, 0)$ , with the values of AL and RD now no longer potential synchronic causes to the action  $F$  that satisfies the shortest distance to goal heuristic (denoted as “\*”) and the still relevant synchronic causes are AA = A3 and RA = 0. Hence after two levels of node expansion, the system no longer needs to expand the nodes corresponding to applying the force in all the other directions. It just needs to use  $F(*, *, A3, 0)$  to head straight to the GOAL.

Suppose now the system finds itself in another START position that has a different absolute angular placement from the GOAL compared to that in the earlier experience. In this situation the  $F(*, *, A3, 0)$  will not work as now  $F$  applied in the  $A3$  absolute direction will not give rise to an  $RA = 0$ . The system therefore carries out the process again as above as described in Fig. 3 and in this second experience, after two levels of node expansion, it discovers, say,  $F(*, *, AX, 0)$  as the optimal way to apply the force at every step to the GOAL. As  $AX$  is not equal to  $A3$ , the absolute angle  $AA$  is then removed as a synchronic cause and the optimal force is  $F(*, *, *, 0)$ , leaving  $RA = 0$  as the only synchronic cause of the optimal force application – i.e., one that satisfies the shortest distance to goal heuristic.

At this stage, the cognitive system has discovered the heuristic that basically says that no matter where the cognitive system is or how far it is from the GOAL, the optimal way to move it to the GOAL is to apply a force that aims directly at the GOAL. Thus, after two experiences and a total of 4 complete node expansions, the cognitive system learns a general causal rule that *obviates all future needs for search* for this spatial movement to goal (with no intervening obstacle) problem.

## 4.2 The Crawling Robot Problem

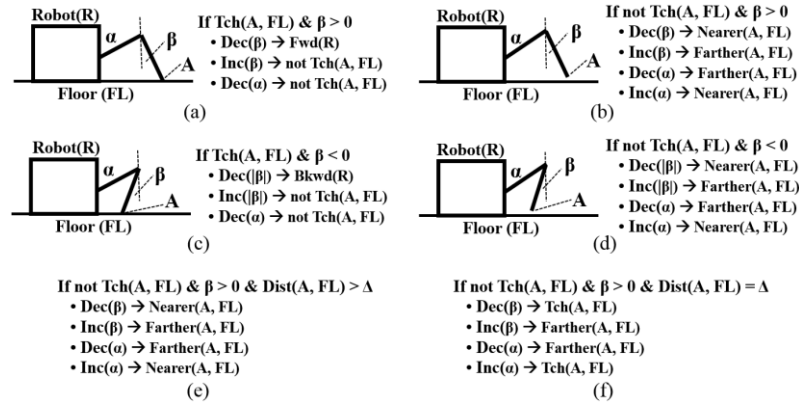
In Fig. 4 we consider a crawling robot problem [6]. Basically, the robot consists of a “body” and two independently movable arms (and their associated angles  $\alpha$  and  $\beta$ ) and the problem is to find a correct sequence of actions of the arms so that the robot will keep moving “forward” or “backward.” 4 elemental actions are available to the robot: Increase/Decrease  $\alpha/\beta$ . This is a situation in which the best-first search combined with the domain independent shortest distance to goal heuristic cannot be used as it is difficult to measure the “distance” between a current state from the goal of, say,  $Fwd(R)^N$  (keeps moving forward – no backward movement at any time). Therefore reinforcement learning is often used for problems such as this [6, 9].

As can be seen from the previous section, there is an early “exploration” phase of the causal learning enhanced search process in which many possibilities are tried (many nodes expanded) and some general and powerful causal rules are discovered that obviate any further extensive search or node expansion altogether. On the other hand, even though there is also an exploration phase in a typical reinforcement learning process [6, 9], there is no attempt to extract the “causes” and powerful generalizations that characterize causal rules or “causal models.” What are learned are basically “state-transition models” with prescriptions of the best courses of actions in various states of the system to achieve a certain goal (e.g.,  $Fwd(R)^N$ ). A huge amount of computation is typically needed in extensive exploration and exploitation phases in reinforcement learning. Reinforcement learning would fair even worse if all contextual factors are included as part of the current “state of the world” which are often removed a priori using domain knowledge. With our effective causal learning process, irrelevant contextual factors (potential synchronic causes) are weeded out in the process of constructing the causal rules as we have described above.

Figs. 4(a), (b), (c), and (d) show different states of the robot arms and to the right of the robot is shown the corresponding general causal rules that govern its behavior in those states. (A computer program that simulates the robot’s behavior would



conceivably have these rules in the program.) For example, in Fig. 4(a), using our parlance, the diachronic cause (action),  $\text{Dec}(\beta)$ , would give rise to a diachronic effect,  $\text{Fwd}(\text{R})$  (i.e.,  $\text{Dec}(\beta) \rightarrow \text{Fwd}(\text{R})$ ) in the presence of the synchronic causes  $\text{Tch}(\text{A}, \text{FL})$  and  $\beta > 0$ .  $\text{Inc}(\beta) \rightarrow \text{not Tch}(\text{A}, \text{FL})$  and  $\text{Dec}(\alpha) \rightarrow \text{not Tch}(\text{A}, \text{FL})$  (causing point A to not touch the Floor) share the same synchronic causes as  $\text{Dec}(\beta) \rightarrow \text{Fwd}(\text{R})$ . Each of  $\text{Dec}(\beta)$ ,  $\text{Fwd}(\text{R})$ , etc. is an event.  $\text{Tch}(\text{A}, \text{FL})$  can be a state or an event (when there is a state change from not Touch to Touch). Similarly for not  $\text{Tch}(\text{A}, \text{FL})$ .



**Fig. 4.** The crawling robot problem. Tch = Touch; Dec = Decrease; Inc = Increase; Fwd = Forward; Bkwd = Backward; Dist = Distance; R = Robot; FL = Floor. A is the dangling end of the distal arm.  $\beta$  has positive values ( $\beta > 0$ ) when the distal arm is swung to the “right” of the vertical dashed line and negative values ( $\beta < 0$ ) when the arm is swung to the “left.”

Hence, unlike “action-state” rules in reinforcement learning, the rules in Fig. 4 do capture the “causal understanding” of the robot’s behavior in its most general form. These rules are learnable using our causal learning method described above, with some minor extensions to the basic algorithm (e.g., handling of parameter ranges such as  $\beta > 0$ ,  $\beta < 0$ , etc.). Once these causal rules are learned in a process of exploration (i.e., move the robot arms about and observe what events occur as a result) they can engender a rapid problem solving process through, say, a backward chaining process, beginning from a desired goal (e.g.,  $\text{Fwd}(\text{R})^N$  – i.e., no  $\text{Bkwd}(\text{R})$  is allowed at any given step - or  $\text{Bkwd}(\text{R})^N$ ), with a drastically reduced search space. Incremental chunking [10] can further reduce the amount of search needed.

In the interest of space, the detailed step-by-step causal learning and problem solving processes will not be described for this case. In the interest of clarity, Fig. 4(b) omits an extra synchronic condition  $\text{Dist}(\text{A}, \text{FL}) > \Delta$ . The complete version is shown in Fig. 4(e).  $\text{Dist}(\text{A}, \text{FL}) = (\text{equal to}) \Delta$  is the condition just prior to a touching event –  $\text{Tch}(\text{A}, \text{FL})$  when A is just slightly above the Floor – and an action  $\text{Dec}(\beta)$  will lead to  $\text{Tch}(\text{A}, \text{FL})$  – the rule in Fig. 4(f). There are two more similar rules corresponding to Fig. 4(d) that are not shown in Fig. 4. There are also a few other rules (e.g., a rule converting  $\beta < 0$  to  $\beta > 0$ ) that are not described here in the interest of space. Suffices it to say that they can all be discovered through the causal learning process. Part of the backward chaining process could make use of the shortest

distance to goal heuristic similar to that discussed in Section 4.1 – that to get a point to touch another point, select the action that brings it *Nearer* the point.

## 5 Conclusion and Future Work

In summary, this paper proposes a method that relies on the effective use of temporal correlation and proximal observation for a process of rapid causal learning that can learn the causal rules that obtain between basic physical and social events and the method is applied to two physical problem solving situations. The learning is carried out in a stage of “exploration” and subsequent problem solving can proceed rapidly with the learned causal rules. This method is necessary to jump start a chain of causal learning processes – it first allows the fundamental physical and social knowledge of the world to be learned and then the learned domain dependent knowledge can subsequently assist the learning of other more complex causal relationships.

In this paper we use parameters (such as location, relative distance, etc.) that are easily available through the sensory input. More complex concepts may come into play as diachronic and synchronic causes. An extension of the basic framework would be to handle probabilistic events.

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